Math 1A Spring 2025
Midterm 1: A Brief Study Guide

March 5, 2025

This study guide is not exhaustive, nor is it necessarily accurate. Please follow the Professor's announcements for updates and the sections to be on the quiz. All of the problems are based on examples or problems from Stewart's book, so it's probably a good idea to read through the text to prepare.

1 Chapter 1: Functions and Foundations

Study Questions:

- Functions: what is a function? How can we represent them?
- What is the domain of a function? What are some basic tricks to find domains?
- What is a piecewise function? Give one example.
- When is a function increasing or decreasing?
- Write examples of polynomials, power functions, rational functions, and trig functions.
- Do you remember the unit circle?
- What are some basic operations we can do on functions?
- What is the composite of functions? What is the inverse of a function?
- Memorize the exponential rules if you haven't already.

Some Practice:

1. Find the domain of $\frac{\cos(x)}{1-\sin^2(x)}$
2. When is $\tan(x)$ increasing? When is it decreasing? When is it neither?
3. Is $\cot(x)$ even, odd, or neither?
4. Let $f(x) = x + 1$ and $g(x) = x^3$. What is $g(f(x))$? What is $f \circ g$? Find the inverse of $f(x)$ and the inverse of $g(x)$. Does x^2 have an inverse function on $\mathbb{R}$?

5. Simplify $\frac{\sqrt{a\sqrt{b}}}{\sqrt[3]{ab}}$. Write the resulting value as $a^x b^y$ for some x and y.
6. What is the range of $\frac{1}{1+e^x}$? What is the domain of its inverse?
7. Find a function $f(x) = A\cos(Bx+C)+D$ for some numbers A, B, C, D so the graph has range $[5, 8]$, period $5/2$, and $f(0) = 13/2$.
8. What is the value $\cos^{-1}(1)$? What about $\tan^{-1}(1)$?

Review Test: Page 68 in the textbook.

2 Chapter 2: Limits and Derivatives

Study Questions:

- How do we define a limit intuitively? How do we define it precisely?
- When given the graph of a function, what do you need to check to evaluate $\lim_{x\to a} f(x)$?
- What should we do when a $\frac{0}{0}$ appears in our limit?
- Give an example where $\lim_{x\to a} f(x)$ diverges to ∞.
- Write out the 5 limit laws. What assumptions do we require for these to hold?
- What does the squeeze theorem say? Give one example where we might apply it.
- Yes, you should know $\epsilon - \delta$ definitions. You should be able to use these to find limits on functions like $4x$, $\sqrt{x}$ (Example 3, page 110) and x^2 (Example 4, page 110)
- What is the definition of continuity? How do we check it? What does it mean to be continuous on an interval? What are the common types of discontinuity?
- Is the sum of two continuous functions also continuous? What about the product?
- Use IVT to find zeroes of a function
- Be able to find the limits at infinity of functions. Give a function where $\lim_{x\to\infty} f(x) = 0$ and another where $\lim_{x\to\infty} f(x) = \infty$. Can you find functions so $\lim_{x\to\infty} f(x) = L$ for any L?
- What is a derivative? What is the motivating geometric property of taking a derivative? Write the two equivalent definitions of a derivative that we use. What is a secant line and how does it relate to a derivative?

- Be able to think of the derivative both as a number and as a function. What are some common cases where the derivative has a restricted domain (doesn't exist at some point)? If the derivative exists at a point, is the function continuous there? Why?

Practice:

1. Evaluate $\lim_{x\to 2\pi^-} x\csc(x)$.
2. Evaluate $\lim_{x\to 1} \frac{x^2-1}{x-1}$.
3. Let $f(x) \le \sec(x)$ for all x. What can we say about $\lim_{x\to \frac{\pi}{2}^-} f(x)$?
4. What is $\lim_{t\to 2}(\frac{t^2-2}{t^3-3t+5})^2$? What limit laws did you use to calculate it.
5. Evaluate $\lim_{x\to\infty} \frac{x^2\sin(x)+3xe^{\frac{1}{x}}}{x^5}$.
6. Use the ϵ-δ definition to show that $\lim_{x\to 1} 4x+2 = 6$. Use it to show $\lim_{x\to 2}\frac{1}{x} = \frac{1}{2}$.
7. Where is the function $f(x) = \frac{\ln(x)+\sec(x)}{x^2-1}$ continuous?
8. Evaluate $\lim_{x\to 1} \sin^{-1}(\frac{1-\sqrt{x}}{1-x})$.
9. Evaluate $\lim_{x\to\pi} \sin(x+\sin(x))$
10. Is there some x so $e^x = -x$? Prove it.
11. Evaluate $\lim_{x\to\infty} \frac{\sqrt{9x^4+2x}}{\sqrt[3]{8x^6+5}}$.
12. Evaluate $\lim_{x\to -\infty} e^{2x+5} + \frac{\sin(5x)}{12x}$
13. Evaluate $\lim_{x\to\infty} x - \sqrt{x^2-1}$.
14. Use the limit definition fo the derivative to differentiate
    (a) $f(t) = \frac{2t+1}{t+3}$
    (b) $f(y) = y^3$
    (c) $g(z) = \frac{4}{\sqrt{1-z}}$
15. Let $f(x) = x\sin(\frac{1}{x})$. Does $f'(0)$ exist?
16. Find a function $f(x)$ so that $f'(x)$ has a vertical asymptote at 0. Find a function $g(x)$ which is differentiable everywhere except $x=3$, but is still continuous.
17. Find the fifth derivative of $f(x) = x^5$.

3 Chapter 3: Differentiation Rules

Study Question:

- What are the derivatives of constants? How do we take the derivatives of powers of x?
- Make a list of the derivative rules as you study.
- What happens when we take the derivatives of sums of functions? What about constant multiples? How do we take the derivatives of polynomials? What rules do we use for these?
- How do we take derivatives of exponentials?
- Write out the product and quotient rule for general functions $f(x)$,$g(x)$
- What are the derivatives of the trig functions $\sin(x), \cos(x)$? What about the extended trig functions $\tan(x), \sec(x), \csc(x), \cot(x)$? Do we need to memorize these or can we derive them?
- Write out the chain rule. What assumptions do we need to apply this rule?
- Why do we use implicit differentiation? Why is it useful?
- Use implicit differentiation to take derivatives of the inverse trig functions.
- What is the derivative of $\ln(|x|)$? What is the derivative of $\log_b(x)$? How can we use log properties to make other derivatives more simple?
- If we make it to related rates (3.9) and Linear approximation (3.10), read those chapters carefully. Related rates is a mostly about critical thinking in the problem-by-problem case. Linear approximation is a big reason we take derivatives at all.

Practice:

1. Find the derivative of $f(x) = x^3 + 3x^{5/2} + 2^x$
2. Differentiate $g(z) = 4z^2e^z \cos(2z)/$
3. Differentiate $h(w) = \frac{3w^2+2}{sin(w)}$. What is the domain of the derivative?
4. Differentiate $f(x) = \sqrt{x + \sqrt{x}}$.
5. Differentiate $x(y) = \sin(\cos(\tan(y)))$
6. Differentiate $k(x) = \tan^2(x)$

7. Find the tangent line to the curve $x^3 + y^3 = 9$ at the point $(1, 2)$.
8. Find y' if $\sin(x + y) = y^2 \cos(x)$
9. Differentiate $\tan^{-1}(x^2 + \sin(x))$
10. Find $\frac{df}{dx}$ for $f(x) = \ln(\frac{x^2+5}{\sqrt{2x-2}})$
11. Differentiate $y = x^{\sqrt{x}}$.
12. Consider a cone whose height and base diameter are always equal. If the volume of the cone increases at a rate of 50 cubic feet per minute, at what rate is the length of the slope of the cone increasing when the cone is 10 feet tall?