Solutions

Spring 2025 MATH 1A

Worksheet: Wednesday 3/5

Exercises:

1. Compute the derivative of the following functions:

   (a) $f(\theta) = 2\sec\theta - \csc\theta$ $\quad f'(\theta) = -2\sec(\theta)\tan(\theta) + \csc(\theta)\cot(\theta)$

   (b) $g(x) = \frac{1-\sec(x)}{\tan(x)} = \frac{\cos(x)-1}{\sin(x)}$ $\quad g'(x) = \frac{-\sin^2(x) - (\cos(x)-1)\cos(x)}{\sin^2(x)}$

   (c) $h(x) = e^{\tan(x)}$ $\quad h'(x) = e^{\tan(x)}\sec^2(x)$

   (d) $f(x) = \sqrt{x^2 + \sin(x)e^x}$ $\quad f'(x) = \frac{2x + \cos(x)e^x + \sin(x)e^x}{2\sqrt{x^2+\sin(x)e^x}}$

   (e) $g(x) = (x^2 + e^{2x-1})^3$ $\quad g'(x) = 3(x^2+e^{2x-1})^2(2x + 2e^{2x-1})$

   (f) $h(y) = ((3x^5 + e^{2x} + x^4\tan(x))^{12} + 2x)^3$

   $h'(y) = 3((3x^5+e^{2x}+x^4\tan(x))^{12}+2x)^2\left[2 + 12(3x^5+e^{2x}+x^4\tan(x))^{11}(15x^4+2e^{2x}+x^4\sec^2(x) + 4x^3\tan(x))\right]$

   (g) $f(\varphi) = \cos\varphi/(1-\sin\varphi)$

   $f'(\varphi) = \frac{(1-\sin(\varphi))(-\sin(\varphi)) + \cos^2(\varphi)}{(1-\sin(\varphi))^2}$

   (h) $g(z) = \cot(z)\cos^2(z)$ $\quad g'(z) = \frac{\sin(z)(3\cos^2(z))(-\sin(z)) - \cos^4(z)}{\sin^2(z)}$

   (i) $h(x) = 2^{\sqrt{\sin(x)}}$ $\quad h'(x) = 2^{\sqrt{\sin(x)}}\ln(2)\cdot\frac{1}{2\sqrt{\sin(x)}}\cdot\cos(x)$

   (j) $f(x) = \cot^2(\sin(x))$ $\quad f'(x) = 2\cot(\sin(x))\csc(\sin(x))\cot(\sin(x))\cos(x)$

   ~~(k)~~ $g(x) = \sin^2(\exp(\sin^2(x)))$

   ~~(l)~~ $h(x) = 2^{3^{4^x}}$

   ~~(m)~~ $f(x) = \sqrt{x + \sqrt{x + \sqrt{x}}}$

   ~~(n)~~ $g(y) = \sqrt{\frac{1+\sin y}{1+\cos y}}$

~~(d)~~

$$h(x) = \sin\left(\frac{e^{x/e^x}\csc(\pi^x)x^{4/5}}{\tan^2\left(12\sin\left(\sqrt{x}^{(1+\sqrt{5})}\right)\right)}\right) \quad \text{Sorry!}$$

2. Find the first and second derivatives of:
   $f'(x) = 4x^3 + 9x^2 + 16 \,;\, f''(x) = 12x^2 - 18x$
   (a) $f(x) = x^4 - 3x^3 + 16x$
   (b) $f(r) = \sqrt{r} + \sqrt[3]{r}$ $\quad f'(r) = \frac{1}{2\sqrt{r}} + \frac{1}{3\sqrt[3]{r^2}} \,;\, f''(r) = -\frac{1}{4r^{3/2}} + \frac{2}{9r^{5/3}}$
   (c) $g(y) = 3e^y - 5y$
   $g'(y) = 3e^y - 5 \,;\, g''(y) = 3e^y$

3. Suppose $f(x) = \sin^2(x)e^{-x}$ and $x(t) = \sqrt{t}/t^2$. Find $f'(x)$ and $x'(t)$. Find

$$\frac{\mathrm{d}f}{\mathrm{d}t} = \frac{\mathrm{d}}{\mathrm{d}t}f(x(t)).$$

$$\frac{df}{dt} = \left(2\sin\left(\frac{\sqrt{t}}{t^2}\right)e^{-\sqrt{t}/t^2} + \sin^2(\sqrt{t}/t^2)\,e^{-\sqrt{t}/t^2}(-1)\right)\left(-\frac{3}{2t^{5/2}}\right)$$

4. Find the 13th derivative of $f(x) = \cos(2x)$. ~~Find the 5th derivative of~~ $\cancel{x^2e^{4x+3}}$.
   $f^{(13)}(x) = 2^{13}(-1)\sin(2x)$

5. For which values of r does $y(x) = e^{rx}$ solve the following differential equation?

$$y'' - 4y' + 3y = 0$$

$y'' = r^2e^{rx}$
$y' = re^{rx}$

$r^2e^{rx} - 4re^{rx} + 3e^{rx} = 0$
$\iff r^2 - 4r + 3 = 0$
$\iff r = 3,\ r = 1$