Exponential Functions

GSI

Math 1A

Math 1A worksheet - 1/31/2025

(1) Simplify each expression as much as possible using rules of exponents

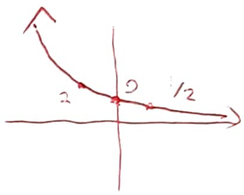
$$\left(\frac{16}{9}\right)^{1/2} = 4/3$$

$$\sqrt[3]{3} \cdot 3^{2/3} = 3$$

$$\frac{2^3 \cdot 3^3}{6^4} = 1/6$$

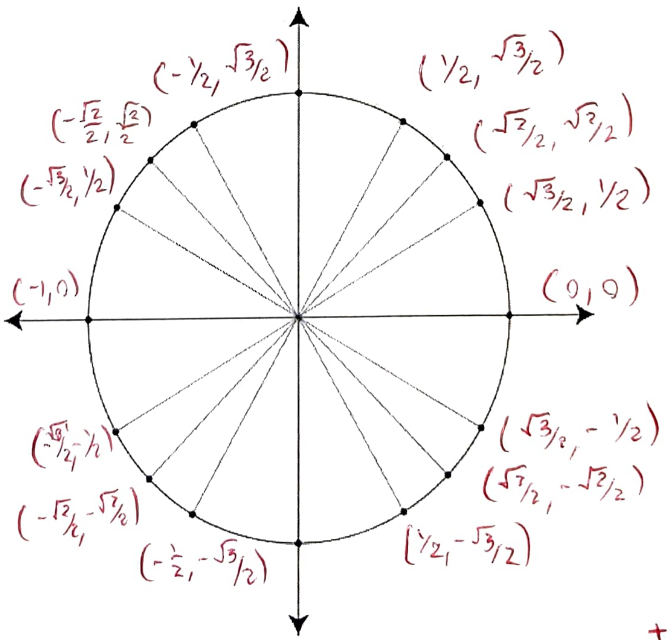
D: $(-\infty, \infty)$

R: $(-2, \infty)$



(2) Find the domain and range of the function $f(x) = \left(\frac{1}{2}\right)^x - 2$. Sketch a graph of $f(x)$ by plotting the points on the graph corresponding to $f(-1), f(0), f(1)$ and using your knowledge of exponential functions.

(3) Label the special angles and the (x, y) coordinates of each point on the unit circle below. (Recall that at a given angle θ, the corresponding point on the unit circle is given by $(\cos(\theta), \sin(\theta))$)



$\rightarrow \pm \pi/6 + 2\pi k$, $\pi/4 + \frac{\pi}{2} k$

$k \in \mathbb{Z}$

Using the values you have added to this circle determine for which values of θ, in radians, we have that $\cos(\theta) = \frac{\sqrt{3}}{2}$. Which values of θ satisfy $\sin^2(\theta) - \frac{1}{2} = 0$?

(4) Is there a function $h(x)$ with domain $\mathbb{R} \setminus \{0\}$ such that $h(h(x)) = x$? What about with domain $\mathbb{R}$?

yes, $h(x) = x$

(5) For every pair of functions f, g given below, list each point in $\mathbb{R}^2$ where the graphs of f and g intersect.

(a)

$$f(x) = x^2 - 3 \qquad g(x) = -2x$$

$x^2 + 2x - 3 = 0$

$(x+3)(x-1)$

$(-3, 6)$

$(1, -2)$

(b)

$$f(x) = \tan(x)^2 \qquad g(x) = 1$$

$\{\pi/4 + \frac{\pi}{2}k \mid k \in \mathbb{Z}\}$

$y = 1$

(c)

$$f(x) = 2^{x-3} \qquad g(x) = 3^x$$

$2^{x-3} = 3^x$

$x - 3 = x \log_2(3)$

$x = \dfrac{3}{1 - \log_2(3)}$ $\quad y = 3^x$

(6) A spherical balloon is being inflated and we wish to model it's surface area over time. Let r denote the radius of the balloon, V denote its volume, and A denote its surface area.
Recall that for spheres $V = \frac{4}{3}\pi r^3$ and $A = 4\pi r^2$.

(a) Find an expression for $V(t)$, the volume of the balloon as a function of time, given that its volume begins at 2 ft^3 and increases at a constant rate of $\frac{1}{2}$ ft^3 per minute. $V(t) = 2 + t/2$

(b) Use your knowledge of spheres to find an expression for $A(V)$, the surface area of the balloon as a function of its volume. ~~$r(t) = [\frac{3}{4\pi}(2 + t/2)]^{1/3}$~~

$A = 4\pi r^2$

$= 4\pi\left(\frac{3}{4\pi}V\right)^{2/3}$

(c) Use function composition to express the surface area of the balloon as a function of time. Simplify your result as much as possible.

$A(t) = 4\pi\left(\frac{3}{4\pi}\right)^{2/3}(2 + t/2)^{2/3}$

$= 3^{2/3}\pi^{1/3}4^{1/3}(2 + t/2)^{2/3}$