Math 1A Spring 2025 2/28

Mini-Lecture

- Product Rule:

$\frac{d}{dx}(f(x)g(x)) = f'(x)g(x) + f(x)g'(x)$ <u>if</u> both derivatives exist

Pf: $\lim_{x \to h} \frac{f(x+h)g(x+h) - f(x)g(x)}{h}$

$= \lim_{h \to 0} \frac{f(x+h)g(x+h) - f(x)g(x+h) + f(x)g(x+h) - g(x)f(x)}{h}$

$= \lim_{h \to 0} g(x+h)\left[\frac{f(x+h)-f(x)}{h}\right] + f(x)\left[\frac{g(x+h)-g(x)}{h}\right]$

Ex: $\frac{d}{dx}(x^2e^x) = 2xe^x + x^2e^x$

- Quotient Rule

$\frac{d}{dx}\left(\frac{f(x)}{g(x)}\right) = \frac{g(x)f'(x) - f(x)g'(x)}{g(x)^2}$ <u>if</u> both derivatives exist <u>and</u> $g(x) \neq 0$.

"low-dee-high minus high-dee-low over low squared"

Pf: $\frac{d}{dx}\left(f(x) \cdot \frac{1}{g(x)}\right) = \frac{f'(x)}{g(x)} + f(x) \cdot \frac{g'(x)}{g(x)}$

or

long limit one.

Ex: $\frac{d}{dx}\left(\frac{x}{1+x^2}\right) = \frac{x(2x) - (1+x^2)}{(1+x)^2} = \frac{x^2-1}{(x^2+1)^2}$

Problems:

① Differentiate

$g(x) = \frac{1+2x}{3-4x}$

$y = \frac{e^x}{1-e^x}$

$f(z) = (1-e^z)(z+e^z)$

$F(t) = \frac{At}{Bt^2+Ct^3}$

(2) Suppose $f(4)=2$, $g(4)=5$, $f'(4)=6$, $g'(4)=-3$

Find $h'(4)$ for

$h(x) = 5f(x) + 3g(x)$

$h(x) = f(x)\,f(x)\,g(x)$

$h(x) = \dfrac{f(x)}{f(x)+g(x)}$

(3) If $f(2)=10$ and $f'(x) = x^2 f(x)$, find $f''(2)$.

(4.) Prove the quotient rule using the limit definition of the derivative

Prove $\frac{d}{dx}\left[\frac{1}{g(x)}\right] = -\frac{g'(x)}{g(x)^2}$ by the quotient rule.