Solutions

Math 1A Spring 2025: 2/26

1. Question 1: Construct a function $f(x)$ which is continuous on $[0, 10]$ and differentiable except at $x = 1$, $x = 5$ and $x = 9$.
   (Hint: use $|x|$) E.g. $f(x) = |x+1| + |x+5| + |x+9|$

2. Question 2: Consider the function $f(x) = (x + 3)^{1/5} + |x - 2|$. Where is $f(x)$ differentiable? Why does $f(x)$ fail to be differentiable at the points it does?
   On $\mathbb{R} \setminus \{-3, 2\}$ -> Vertical asymptote at $x = -3$, Sharp turn at $x = 2$

3. Question 3: Find the derivatives of the following functions:
   (a) $f(x) = e^5$ $f'(x) = 0$
   (b) $f(x) = 3x^2 + 2x + 1$ $f'(x) = 6x + 2$
   (c) $f(x) = \frac{1}{\sqrt{x}}$ $f'(x) = -\frac{1}{2}x^{-\frac{3}{2}}$
   (d) $f(x) = x^e + e^x$ $f'(x) = ex^{e-1} + e^x$

4. Question 4: Evaluate $\lim_{x \to 1} \frac{x^{100} - 1}{x - 1}$. $f(x) = x^{100}$, $\lim = f'(1) = 100$

5. Question 5: Find the tangent line to $f(x) = |\sqrt{x} - 3|$ at $x = 16$. $f'(16) = 1/8$ $(16, 1)$
   $y = \frac{1}{8}(x - 16) + 1$

6. Question 6: Determine whether the following are true or false. If it is true, explain why. If not, give an example that disproves the statement.
   (a) $\lim_{x \to 2} \frac{x}{x-2} - \frac{3}{x-2} = \lim_{x \to 4} \frac{x}{x-2} - \lim_{x \to 4} \frac{3}{x-2}$ False (RHS not defined)
   (b) $\frac{x^2 - 14}{x - 4} = x + 4$ False -> at $x = 4$
   (c) $\frac{d}{dx}(f(x) + g(x)) = f'(x) + g'(x)$ for all f and g. False - if $f(x)$ or $g(x)$ isn't differentiable at a point.
   e.g. $f(x) = \frac{1}{x}$, $g(x) = -\frac{1}{x}$