Solutions

February 23, 2025

Math 1A Worksheet #13

Name: ______________________________

1. Use the squeeze theorem to prove that

$$\lim_{x\to 0} \exp\left(-\frac{1}{|x|}+100\right)x^2 = 0.$$

Hint: when x is really close to 0, then $-\frac{1}{|x|}$ is very negative, so $-\frac{1}{|x|}+100$ is a negative number.

- For $|x| < \frac{1}{100}$, $-\frac{1}{|x|}+100 < 0$ so $0 \le \exp\left(-\frac{1}{|x|}+100\right) \le 1$

$\Rightarrow \lim_{x\to 0} 0 \le \lim_{x\to 0} \exp\left(-\frac{1}{|x|}+100\right)x^2 \le \lim_{x\to 0} x^2 = 0$

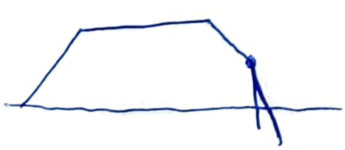
2. The following piecewise defined function defines the position $P(t)$ of a particle as a function of time t.

$$P(t) = \begin{cases} 2t & 0 \le t < \frac{5}{2} \\ 5 & \frac{5}{2} \le t < 7 \\ -t+12 & 7 \le t < 10 \\ -3t+32 & t \ge 10 \end{cases}$$

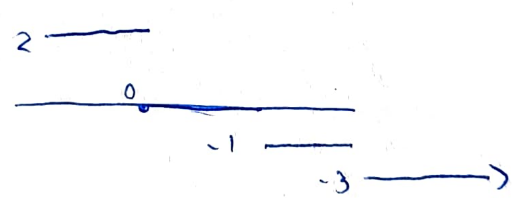
(a) Sketch a graph of $P(t)$.

(b) Sketch a graph of the *velocity function* of the particle.

A.)



B.)



3. Find the slope of the tangent line to the curve at $x = 1$ for each of the following graphs:

(a) $y = 3x^2 + x - 5$ $\quad y' = 6x+1$; 7

(b) $y = x^3 - 2x$ $\quad y' = 3x^2 - 2$; 1

(c) $y = \sqrt{x}$ $\quad y' = \frac{1}{2\sqrt{x}}$; $1/2$

(d) $y = \frac{1}{x+1}$ $\quad y' = \frac{-1}{(x+1)^2}$; $-1/4$

4. Let $f(x)$ be a function, and let C be a real number. Let $g(x) = f(x) + C, h(x) = C$. Let a be another real number.

(a) Check that the derivative $h'(a)$ is equal to 0. $\lim_{x \to a} \frac{c-c}{x-a} = \lim_{x \to a} 0 = 0$

(b) Check that $f'(a) = g'(a)$.

$g' = f' + h' = f'$

6. Kaladin throws a ball into the air on the mysterious planet Roshar at a velocity of 55 ft/s, starting from a height of 6 ft. The height in feet after t seconds is given by $y = -12t^2 + 55t + 6$.

(a) What is the velocity at $t = 2$? $y' = -24t + 55$; 7

(b) At what time will the ball hit the ground? $\frac{-55 + \sqrt{55^2 + 4(12)(6)}}{24}$ Quad. formula

(c) What is the velocity at which the ball hits the ground? ≈ 4.6894 seconds

(You may use calculator for (b), (c)).

-57.5456 ft/s

Solutions

Math 1A Spring 2025 Quiz 3

Name:

1. Determine the limit

$$\lim_{x\to 2} \frac{x^2+2x-8}{x^2-5x+6}. = \lim_{x\to 2} \frac{(x+4)\cancel{(x-2)}}{\cancel{(x-2)}(x-3)} = -6$$

2. Evaluate the limit, if it exists

$$\lim_{h\to 0} \frac{\frac{1}{(x+h)^2}-\frac{1}{x^2}}{h}. = \lim_{h\to 0} \frac{1}{h}\left(\frac{x^2-(x^2+2xh+h^2)}{x^2(x+h)^2} \right)$$

$$= \lim_{h\to 0} \frac{-2x-h}{x^2(x+h)^2} = \frac{-2}{x3} \quad \text{if } x \neq 0 \quad (\text{DNE if } x=0)$$

3. Prove that

$$\lim_{x\to 0} x^4 \cos\frac{2}{x} = 0.$$

$-x^4 \le x^4 \cos(2/x) \le x^4$ and squeeze!

4. Use the Intermediate Value Theorem to show that there is a root of the given equation in the specified interval:

$$e^x = 3-2x, \quad (0,1)$$

$f(x) = e^x - 3 + 2x$ $\quad$ $f(x)$ is continuous on $(0,1)$ ✓

$f(0) = 1-3 = -2 < 0$

$f(1) = e+2-3 > 0$

IVT $\Rightarrow$ there exists $x_0 \in (0,1)$ so $f(x_0) = 0$

and $e^{x_0} = 3-2x_0$