Worksheet 2/20

1.) Why? $-\frac{1}{x}-\frac{1}{x^2} \leq \frac{\sin(x)x+\cos(x)}{x^2} \leq \frac{1}{x}+\frac{1}{x^2}$

So $\lim_{x\to\infty} \frac{\sin(x)}{x}+\frac{\cos(x)}{x^2}=0.$

2.) $f'(x)=\lim_{h\to 0}\frac{\frac{1}{x+h+1}-\frac{1}{x+1}}{h}$

So $f'(0)=\lim_{h\to 0}\frac{\frac{1}{h+1}-\frac{1}{1}}{h}=\lim_{h\to 0}\frac{1}{h}\left(\frac{-h}{h+1}\right)=\lim_{h\to 0}-\frac{1}{h+1}=-1$

$y-f(0)=f'(0)(x-0)$

So $y=1+(-1)(x)$

3.) $f'(x)=\lim_{h\to 0}\frac{\frac{1}{x+h+1}-\frac{1}{x+1}}{h}=\lim_{h\to 0}\frac{1}{h}\left(\frac{-h}{(x+1)(x+h+1)}\right)=-\frac{1}{(x+1)^2}.$

A.) $x\neq -1$

B.) $-\frac{1}{(x+1)^2}$

4.) A.) Sum of limits is the limit of sums

B.) $\lim_{k\to 0}\frac{h(x+k)-h(x)}{k}=\lim_{k\to 0}\frac{c(f(x+k)-f(x))}{k}=c\cdot\lim_{k\to 0}\frac{f(x+k)-f(x)}{k}$

$=c\cdot f'(x)$

5.) $g(x)=5f(x)+6h(x)$

$f'(1)=3(1)^2=3$

$h'(1)=-1/(1)^2=-1$

$g'(x)=9$

6.) i) Yes, $h(x) = |x-2|$

ii.) No, differentiability Implies Continuity.