MATH 1A Worksheet (Feb 19th)

1. Consider the piecewise function

- (a) Does there exist an input x such that $f(x) = \frac{1}{2}? N_{o}$,
- (b) Is it a counter example to Intermediate Value Theorem? Why or why not? No, f(x) is discontinuous.
- 2. (a) Is there a real number x such that it is exactly 1 less than its cube root?
 Yo, apply IVT +0 3√x-(-x on (-6,-1))
 (b) Is there a nonnegative real number x such that it is exactly 1 less than its square
 - (b) Is there a nonnegative real number x such that it is exactly 1 less than its square root? No, $\sqrt{x} 1 < x \neq x \ge 0$.

Is there a nonnegative real number x such that it is more than 1 less than its square root? (that is $x < \sqrt{x} - 1$). Try to argue using results from part(b) and Intermediate Value Theorem.

- 3. (a) If a continuous function f(x) on [0,1] satisfies that $0 \le f(x) \le 1$. Does there exist $0 \le x \le 1$ such that f(x) = x? Yes, $I \lor T$ on f(x) = f(x) x.
 - (b) Consider the function

 $f(x) = \begin{cases} \frac{\pi}{8} & x \text{ is rational, that is of the form } \frac{p}{q} \text{ with some integer } p, q \\ \frac{1}{2}x & \text{otherwise} \end{cases}$

Is this function continuous on [0, 1]?(Hint: use part(a)) No, (x) isn't varional.

(c) Is the function f in part(b) continuous at $x = \frac{\pi}{4}$? **Yes**.

- 4. In this problem you are allowed to use calculator(whether graphing or not) to estimate the limit, except for last part
 - (a) Calculate $\lim_{x \to +\infty} \frac{x^2}{x^3+1}$, $\lim_{x \to +\infty} \frac{x^4+x^2+50000}{x^8+1}$, $\lim_{x \to +\infty} \frac{100000x^4+x^3}{x^{10}+1}$. Did you see any pattern? O, O, O
 - (b) Calculate $\lim_{x \to +\infty} \frac{x^4}{x^3+1}$, $\lim_{x \to +\infty} \frac{x^{10}+x^3}{x^5+80000}$, $\lim_{x \to +\infty} \frac{x^8}{1000x^2+x}$. Any pattern? \swarrow , \backsim
 - (c) Calculate $\lim_{x\to+\infty} \frac{3x^2}{2x^2+1}$, $\lim_{x\to+\infty} \frac{2x^3+10000}{x^3+x}$, $\lim_{x\to+\infty} \frac{2x^4+1}{2x^4+600000x}$. Any pattern? $\mathcal{F}_{\mathcal{F}}$, $\mathcal{F}_{\mathcal{F}}$,
 - (d) Prove the patterns you observed in (a)-(c). Pivicle by highest order term.