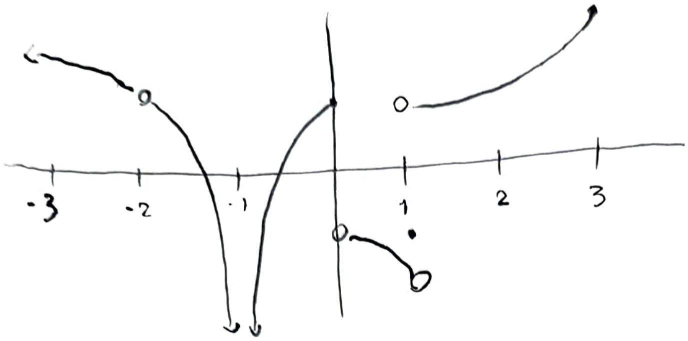
2/14 Discussion

1.)



State intervals where the function is continuous.

$(-\infty,-2)\cup(-2,-1)\cup(-1,0)\cup(0,1)$
$\cup(1,3)$

2.) $f(x) = \begin{cases} \dfrac{x^2-x}{x-1} & \text{if } x \neq 1 \\ 1 & \text{if } x = 1 \end{cases}$

Is $f(x)$ continuous at $x=1$? Use the definition of continuity.

Yes, $\lim_{x\to 1} f(x) = \lim_{x\to 1} x\left(\frac{x-1}{x-1}\right) = 1 = f(1)$.

3.) $f(x) = \dfrac{x^3-8}{x^2-4}$ What should $f(2)$ be so f is continuous?

$f(2) = 3$

4.) Show that there is some $x \in (2,3)$ so $\ln(x) = x - \sqrt{x}$.

$\ln(2) > 2 - \sqrt{2}$ - Both functions are continuous, so apply IVT.

$\ln(3) < 3 - \sqrt{3}$

5.) Show that $f(x)$ is continuous at a if and only if $\lim_{h\to 0} f(a+h) = f(a)$.

$\lim_{h\to 0} f(a+h) = \lim_{x\to a} f(x)$

6.) Show that $f(x) = x^3$ is continuous using the ε-δ definition of continuity.

If $|x-a|<1$, $|x^3-a^3| = |x-a|\cdot|x^2-2a+a^2|$
$\leq |x-a|\cdot(3a^2+4|a|+1)$

So for $\delta = \min\{1, \varepsilon/(3a^2+4|a|+1)\}$, $|x-a|<\delta \Rightarrow |x^3-a^3|<\varepsilon$.

Then, $\lim_{x\to a} x^3 = a^3$, so f is continuous.