GSI

Spring 2025 MATH 1A

Worksheet: Monday 2/10

Exercises:

1. A runner's position in meters as a function of seconds in the first five seconds of running can be modeled by the function $f(t) = t^2 + 4t$, $0 \le t \le 5$. Suppose we want to approximate how fast the runner is moving at $t = 2$ seconds. To do this, we can calculate some average velocities:

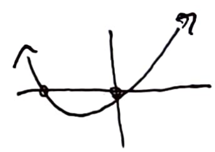
   (a) Calculate the runner's average velocity on the interval $[2, 3]$ $\frac{21-12}{3-1} = 9/2$

   (b) Calculate the runner's average velocity on the interval $[2, 2.5]$ (you may use a calculator)

   (c) Calculate the runner's average velocity on the interval $[2, 2.1]$ (you may use a calculator)

   (d) Using your three previous answers, approximate the runner's instantaneous velocity at $t = 2$. 8

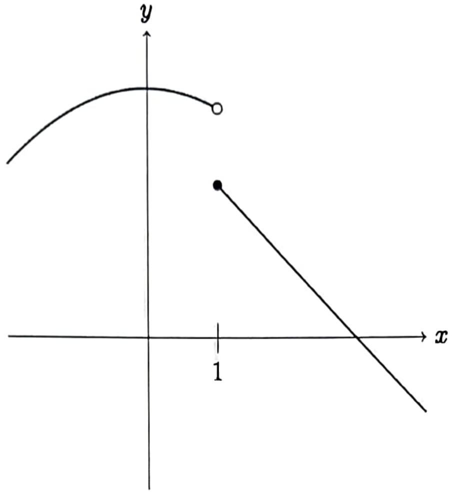
   (e) Draw a graph of f and interpret your last four answers graphically.



   (f) Bonus: Can you represent the instantaneous velocity at $t = 2$ as a limit?

$$\lim_{h \to 0} \frac{f(2+h) - f(2)}{h} = \cancel{\lim_{h \to 0}} \; \cancel{(2+h+h^2)+4}$$
$$= \lim_{h \to 0} \frac{8h + h^2}{h}$$

2. Consider the graph of the following function



(a) Does the limit as $x \to 1$ of this function exist? No

(b) What about as $x \to 1^+$?

(c) What about as $x \to 1^-$?

3. Evaluate the following limits:

(a) $\lim_{x\to 0} \frac{\cos(2x)}{1+2\sin(x)}$ $= 1$

(b) $\lim_{x\to 5} \frac{x^2-6x+5}{x-5}$ (Hint: Try factoring!) 4

(c) $\lim_{x\to 5} \frac{x^2-5x+6}{x-5}$ DNE

(d) $\lim_{x\to 0^+} e^x \ln(x)$ $= -\infty$ (Diverges to $-\infty$)

(e) $\lim_{x\to \pi^+} \frac{x^2}{\sin(x)}$ Diverges to $-\infty$

(f) $\lim_{t\to 0} \frac{\sqrt{1+t}-\sqrt{1-t}}{t}$ $= \lim_{t\to 0} \frac{(1+t)-(1-t)}{t(\sqrt{1+t}+\sqrt{1-t})} = \lim_{t\to 0} \frac{2}{\sqrt{1+t}+\sqrt{1-t}} = 2$

Solutions

Math 1A Spring 2025 Quiz 2

Name:

$12-2=10$ Range: $[2,10]$ Amplitude: 5

1. At Hopewell Cape, the water depth at low tide is about 2 meters and at high tide is about 12 meters. The natural period of oscillation is 12 hours and today high tide occured at midnight. Find a function involving the cosine function that models the water depth $D(t)$ (in meters) as a function of time t (in hours after midnight) on that day.

$\frac{2\pi}{c}=12$

$\Rightarrow c=\pi/6$

$5\cos(\pi/6 x)+7$

2. Express the function $v(t)=\sec(t^2)\tan(t^2)$ in the form $f\circ g$.

$f(t)=\sec(t)\tan(t)$

$g(t)=t^2$

3. If $f(x)=5^x$, show that

$$\frac{f(x+h)-f(x)}{h}=5^x\left(\frac{5^h-1}{h}\right).$$

$$5^x\left(\frac{5^h-1}{h}\right)=\frac{5^{x+h}-5^x}{h}=\frac{f(x+h)-f(x)}{h}$$

4. Find a formula for the inverse of the function

$$y=\frac{1-e^{-x}}{1+e^{-x}}.$$

$y(1+e^{-x})=y+ye^{-x}=1-e^{-x}$

$e^{-x}(y+1)=1-y$

$x=-\ln\left[\frac{1-y}{1+y}\right]$